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LETTER TO THE EDITOR

Ballistic electronic conductance of two parallel channels

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Abstact. For the ballistic motion of a two dimensional non-interacting electron gas at the Fermi momentum k we compute the quantum mechanical conductance (at zero temperature) of two narrow parallel channels of length L and lateral dimensions a and b. The basic global structure of the conductance reveals quantised plateaux of integer multiples of $2e^2/h$. The onset of each step occurs whenever one of the channels can populate a new subband. In the special cases where the relation $k|a-b| = n\pi$ ($n = 0, 1, 2, \ldots$) holds, the quantised steps are of two units. Experimentally, $k|a-b|$ can be determined with some uncertainty (about 0.2π) around certain values. Taking these uncertainties into account in our calculations, we find that the quantised conductance pattern is somewhat smeared and the two unit jumps become more profound. A closer inspection of the theoretical curves shows an oscillatory behavior which results mainly from longitudinal wave resonances in each channel, but interference effects between the two channels may also be present.

Recent experiments (Wharam *et al* 1988a, b, van Wees *et al* 1988) on the ballistic motion of electrons through a narrow channel revealed the phenomenon of conductance quantisation, i.e. the conductance increases as a function of the channel width by integer values in units of the fundamental conductance unit. This fascinating phenomenon has been the subject of much theoretical work (Khmel'nitskii and Shekhter 1988, Haanappel and van der Marel 1989, Glazman and Lesovick 1988, Kirczenow 1988, Avishai and Band 1988, Szafer and Stone 1989, Landauer 1988), and opened the road for the analogy between ballistic motion of electrons in confined regions and the theory of waveguides. A natural question which arises and has recently been tested experimentally (Smith *et al* 1989), is what happens when there is more than one channel. This corresponds classically to the parallel addition of two (or more) resistors and it is of great interest to check whether an analogous quantum-mechanical law holds.

The purpose of this Letter is to present the results of our exact quantum mechanical calculations of the conductance of two parallel channels. We are mainly interested here in the global behaviour and not with the finer dependence on precise geometry, and hence we study the easiest configuration in which the channels are not tapered. The basic quantity we calculate is the transmission amplitude matrix $\mathbf{t}$, from which the conductance is evaluated using the linear conductance formula, $G = (2e^2/h)\mathrm{Tr}(\mathbf{t}\mathbf{t}^+)$ (Fisher and Lee 1981, Lee and Fisher 1981). For the case of ballistic motion we show that the calculated conductance behaves globally according to the classical law of addition of

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conductances in parallel. The onset of each step occurs whenever one of the channels can populate a new subband irrespective of the second channel. Superposed on this global background there is a rich oscillatory structure which is related partly to the longitudinal wave resonances in each channel and partly to the interference of waves emanating from the two channels, as in a two-slit experiment. Our calculational techniques can be easily extended to study samples containing impurities and the effect of a perpendicular magnetic field (Avishai and Band 1989).

The geometry in the present problem is shown in the inset of figure 1. Consider the quantum mechanical motion of a particle with mass m and (Fermi) energy E in a planar region composed of two semi-infinite strips defined by $(-\infty < x \leqslant 0, 0 \leqslant y \leqslant w)$, $(L \leqslant x < \infty, 0 \leqslant y \leqslant w)$ and a finite domain with two channels of widths a and b which are defined by $(0 < x \leqslant L, 0 \leqslant y \leqslant a$ and $w - b \leqslant y \leqslant w)$. We look for a solution of the Schrödinger equation, $-\Delta\Psi_n = (2mE/\hbar^2)\Psi_n$, corresponding to an incoming initial wave moving from left to right in a definite channel n, with hard wall boundary conditions. The wavefunction in the four respective regions has the form

$$\Psi_n(x, y) = \sqrt{\frac{2}{w}}\left[\mathrm{e}^{\mathrm{i}k_n x} \sin\left(\frac{n\pi y}{w}\right) + \sum_{m-1}^{M} R_{mn}\, \mathrm{e}^{-\mathrm{i}k_m x} \sin\left(\frac{m\pi y}{w}\right)\right] \qquad (-\infty < x \leqslant 0, 0 \leqslant y \leqslant w) \tag{1a}$$

$$\Psi_n(x, y) = \sqrt{\frac{2}{w}} \sum_{m=1}^{M} T_{mn}\, \mathrm{e}^{\mathrm{i}k_m(x-L)} \sin\left(\frac{m\pi y}{w}\right) \qquad (L \leqslant x < \infty, 0 \leqslant y \leqslant w) \tag{1b}$$

$$\Psi_n(x, y) = \sqrt{\frac{2}{a}} \sum_{j=1}^{J_a} (s_m\, \mathrm{e}^{\mathrm{i}q_j x} + t_{jn}\, \mathrm{e}^{-\mathrm{i}q_j x}) \sin\left(\frac{j\pi y}{a}\right) \qquad (0 < x \leqslant L, 0 \leqslant y \leqslant a) \tag{1c}$$

$$\Psi_n(x, y) = \sqrt{\frac{2}{b}} (u_{jn}\, \mathrm{e}^{\mathrm{i}p_k x} + v_{jn}\, \mathrm{e}^{-\mathrm{i}p_j x}) \sin\left(\frac{j\pi(y + b - w)}{b}\right) \qquad (0 < x \leqslant L, 0 \leqslant y \leqslant a) \tag{1d}$$

Here, $k^2 = 2mE/\hbar^2$, and the wave numbers $k_n q_j$ and p_j are given by

$$k_n = \sqrt{k^2 - n^2\pi^2/w^2} \qquad q_j = \sqrt{k^2 - j^2\pi^2/a^2} \qquad p_j = \sqrt{k^2 - j^2\pi^2/b^2}. \tag{2}$$

Thus, the reflection and transmission amplitudes R_{mn} and T_{mn} $(m, n = 1, 2, \ldots, M)$ are elements of finite-dimensional matrices, the number M contains N channels for which the momenta k_n of equation (2) are real (namely $n \leqslant N = [kw/\pi]$) ($[x]$ meaning integer values of x) plus a finite number of $M - N$ of evanescent waves for which the momenta k_n are imaginary which is sufficient to guarantee convergence with desired accuracy. The role of the evanescent waves within the channels is equally important. Therefore, we set $J_a > [ka/\pi]$ and $J_b > [kb/\pi]$ in equations (1c) and (1d) respectively and fix them so that convergence is assured.

Matching the wavefunction and its derivative (with respect to x) at $x = 0, L$ and using the completeness of the sine functions in each pertinent interval we get four sets of equations for the unknown coefficient matrices s_{jn}, t_{jn}, u_{jn} and v_{jn}. With these coefficients

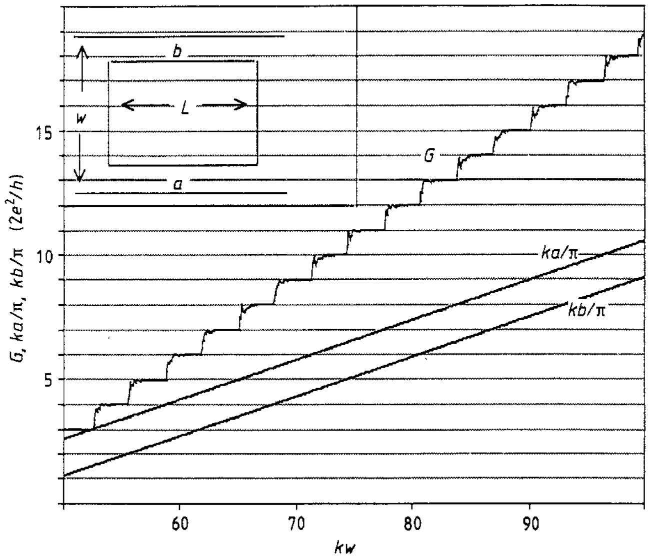

Figure 1. Conductance G (in units of $2e^2/h$) of two parallel channels of length L and widths a, b connected to wires of width w (layout shown in inset) for a two-dimensional electron gas with Fermi momentum $\boldsymbol{k}$. The conductance is plotted as a function of kw between 50 and 100, for $kL = 200$. The relation between a, b and w is given in equation (7).

at hand we can easily evaluate the transmission and reflection amplitude matrices T_{mn} and R_{mn}. Flux-normalised reflection and transmission amplitudes and their unitarity relations read

$$r_{mn} = \sqrt{k_m/k_n}\, R_{mn} \qquad t_{mn} = \sqrt{k_m/k_n}\, T_{mn} \tag{3}$$

$$\sum_{m=1}^{N} (|r_{mn}|^2 + |t_{mn}|^2) = 1 \qquad n = 1, 2, \ldots N. \tag{4}$$

Before entering into finer details, we stress that our results indicate that the system of two orifices behave as a classical system of two conductors added in parallel, and that an approximate formula for the conductance which just counts the number of filled subband in each channel can be used, namely

$$G = 2(e^2/h)\{[ka/\pi] + [kb/\pi]\}. \tag{5}$$

This is indeed the case as far as the quantised plateaux are concerned, as we easily verify in our more refined numerical simulations. A key quantity in this context is the difference parameter

$$\alpha = k(a - b)/\pi \tag{6}$$

which determines the distance between two successive jumps. If $\alpha = n + \frac{1}{2}$ (n being an integer) there are clearly separated jumps of one unit, while if $\alpha = n$, the jumps are of two units. However, as has been recently realised (Kirczenow 1988, Avishai and Band 1988, Szafer and Stone 1989), the quantised plateaux are not the only feature of conductance of narrow channels, since it has also a very rich oscillatory structure. Therefore, it is important to study the detailed behaviour (that is, beyond the approximation (5)), for the two channel configuration. We evaluated $\mathrm{Tr}(\mathbf{tt}^+)$ for channels of dimensionless length parameter fixed at $kL = 200$ connected to wires with varying (dimensionless) width $50 < kw < 100$. The relation between the width of the wires and the widths of the channel can be chosen in various ways in order to test several features. Our first chice (shown in figure 1) for which $\alpha = 1.5$ and which covers aspect ratios between 6 and 60 is

$$ka = kw/2 - 17 \qquad kb = ka - 1.5\pi. \tag{7}$$

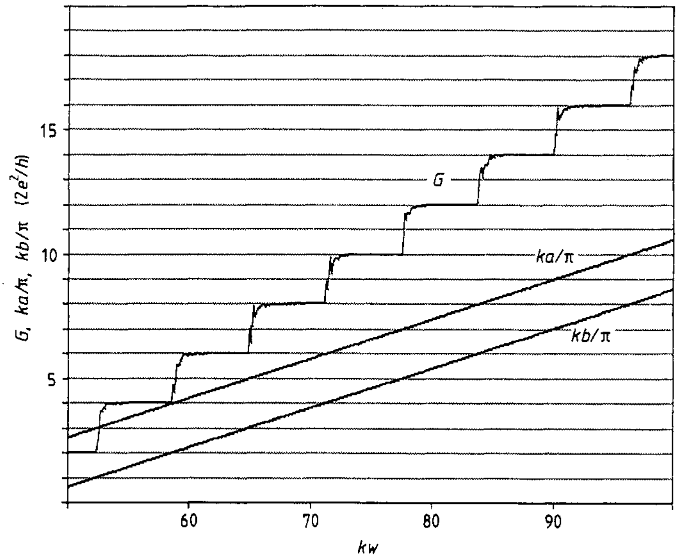

Figure 2. As figure 1 but the relation between a, b and w is given in equation (8).

This choice of parameters enables us to study several aspects: (i) The subbands in the two channels are populated one after the other with equal distance between them. This leads therefore to well defined steps of one unit of conductance and the law of addition is almost exact. (ii) When $kw = 50$, the smaller channel is almost closed for propagating waves, and the conductance changes whenever the other channel populates subband. (iii) For $50 < kw < 60$ when the channels are narrow and far from each other, we can check their independent contributions to the conductance. This is manifested by the structure of the conductance at small w, composed of a steep raise followed by oscillations, which is typical to its structure in a single channel configuration (Avishai and Band 1988). (iv) For $90 < kw < 100$, the two channels cover more than 60% of the strip and we expect interference effects to be significant. This is clearly seen as we proceed from one conduction step to the other. The oscillations start before the steep raise and their structure becomes richer. In the single-channel configuration, the structure of all steps is nearly identical. This is the scaling behavior conjectured recently (Szafer and Stone 1989), and the slight difference is due to the change in the aspect ratio. Here, the difference is more pronounced due to both aspect ratio and interference effects.

Our second choice of parameters for which $\alpha = 1.926$ (namely, close to the integer 2) is

$$ka = 8.157 + 0.5(kw - 50) \qquad kb = 2.105 + 0.5(kw - 50). \tag{8}$$

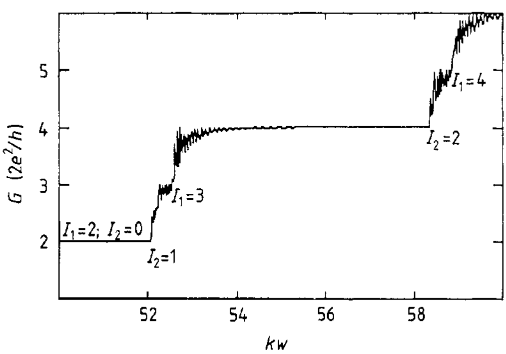

Figure 3. As figure 2, focusing on the first interval $50 \leqslant kw \leqslant 60$. $I_1 = [ka/\pi]$, $I_2 = [kb/\pi]$.

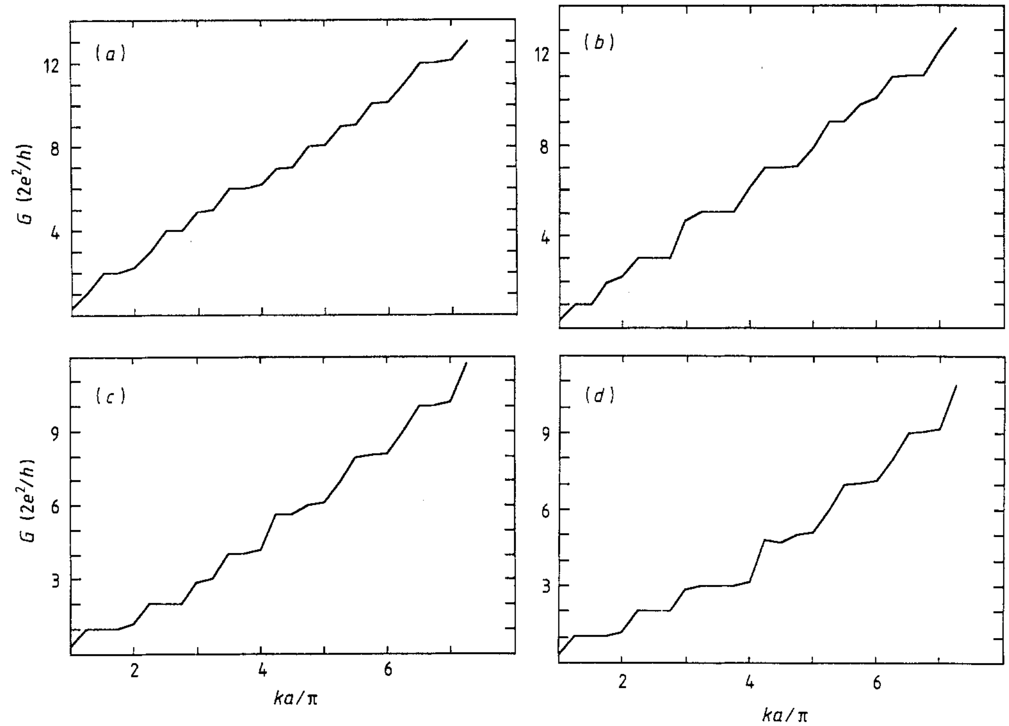

Figure 4. Conductance of two parallel channels of length $kL = 100$ and widths a, b connected to wires of width $kw = 100$ for a two-dimensional electron gas with Fermi momentum k. The conductance is plotted as a function of $1 < ka/\pi < 6$ (a) with a difference parameter $\alpha = k(a-b)/\pi = 0.4$ with uncertainty 0.2; (b) with $\alpha = 0.8$; (c) with $\alpha = 2.3$; (d) with $\alpha = 3.3$.

The essential difference here is that now the subbands in the two channels are populated almost simultaneously since $k(a-b) \simeq 2\pi$ and the expectation is for steps of two units of conductance. This is indeed the case as we can see from figure 2. Since $ka - kb$ is slightly different from an integer multiple of π this enables us to follow the combination of oscillations which follow the raise due to population of a new subband in the first channel and the immediate successive raise due to population of a new subband in the second channel. This is illustrated in figure 3 for the narrow channels ($50 \leqslant kw \leqslant 60$).

Our main goal is to study the conductance with parameters close to the experimental ones. We then fixed the value of kw to be large enough so that it will be greater than the experimental value of $k(a+b)$. The value of the difference parameter was given four experimental values (Smith *et al* 1989a, b): 0.4, 0.8, 2.3 and 3.3, all with uncertainty of 0.2. The gate voltage in each experiment varied in such a way that the first channel was given the values $1 < ka/\pi < 6$, with resolution 0.2. The results are shown in figures 4(a)–4(d), and the dominant features can be summarised as follows: (i) For $\alpha = 0.4$ and 0.8, (figures 4(a) and 4(b)), the conductance undergoes jumps of both 1 and two units, but the latter ones are more pronounced (although neither of them are sharp). (ii) For $\alpha =$ 2.3 and 3.3 (figures 4(c) and 4(d)) there are definite jumps of one unit at the beginning, and after that, the structure resembles the one discussed above for the case (i). The one unit jumps at the beginning are clearly the result of an occurrence of a very narrow channel which cannot have one-dimensional subbands, and the other channel is filled as if the system consists of the wider channel alone.

Our exact calculations of the conductance are in general agreement with the observations (Smith *et al* 1989a, b). In particular, we confirm the clear one unit conductance jump for high gate voltages and for the case where $\alpha = k(a-b)/\pi = 2.3$ and 3.3. We find that the structure is dominated by two unit conductance jumps. Beyond that, the structure found experimentally for these two cases ($\alpha = 2.3$ and 3.3, figures 4(*c*) and 4(*d*)) may indicate a smearing of the one unit conductance jumps. Later these seem to disappear completely in the experimental results for $\alpha = 0.4$ and 0.8 but in the theoretical calculations, illustrated in figures 4(*a*) and 4(*b*), they still show up, (albeit weakened). More work is needed in order to decide whether this discrepancy is fundamental.

In conclusion, the study of the conductance of two channels in parallel indicates that the classical law of addition holds almost exactly. Smearing effects tend to weaken the one unit conductance jumps yielding a structure which is dominated by two unit conductance jumps. The oscillatory structure becomes richer than in the case of the one channel case due to interference. Experimental confirmation of these results imply that electronic devices operating with ballistic electrons in confined regions can be designed to have characteristics similar to analogous waveguide devices, e.g., filters, couplers, resonators, etc.

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